basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

NATIONAL SENIOR CERTIFICATE

GRADE 12

MATHEMATICS P1

FEBRUARY/MARCH 2013

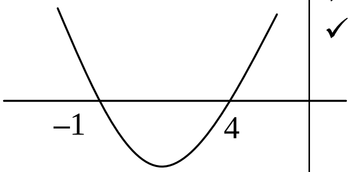
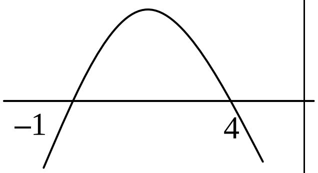
MEMORANDUM

MARKS: 150

This memorandum consists of 19 pages.

QUESTION 1

1.1.1	$(x^2-9)(2x+1)=0$ $(x-3)(x+3)(2x+1)=0$ $x=\pm 3$ or $x=-\frac{1}{2}$ **OR** $(x^2-9)(2x+1)=0$ $x=\pm 3$ or $x=-\frac{1}{2}$	✓ $(x-3)(x+3)$ ✓ ± 3 ✓ $-\frac{1}{2}$ (3) ✓ -3 ✓ 3 ✓ $-\frac{1}{2}$ (3)
1.1.2	$x^2+x-13=0$ $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ $=\frac{-1\pm\sqrt{1-4(1)(-13)}}{2}$ $=\frac{-1\pm\sqrt{53}}{2}$ $x=3{,}14$ or $x=-4{,}14$	✓ subs into formula ✓ $\sqrt{53}$ ✓answer ✓answer (4)
1.1.3	$2\cdot 3^x=81-3^x$ $2\cdot 3^x+3^x=81$ $3^x(2+1)=81$ $3^x=27$ $3^x=3^3$ $x=3$ **OR** $2.3^x=81-3^x$ $2.3^x+3^x=81$ $3^x(2+1)=81$ $3^{x+1}=3^4$ $x+1=4$ $x=3$	✓ $2\cdot 3^x+3^x=81$ ✓ 3^x as common factor ✓ simplification ✓ answer (4) ✓ $2\cdot 3^x+3^x=81$ ✓ 3^x as common factor ✓ $3^{x+1}=3^4$ ✓ answer (4)

1.1.4	$(x+1)(4-x)>0$ $(x+1)(x-4)<0$ + 0 − 0 + –1 4 or  $-1<x<4$ **OR** $(x+1)(4-x)>0$ – 0 + 0 – –1 4  $-1<x<4$	✓ change of sign ✓ both critical values ✓ correct inequality sign (3) ✓ method ✓ both critical values ✓ correct inequality sign (3)
1.2.1	$2^x+2^{x+2}=-5y+20$ $2^x(1+2^2)=-5y+20$ $2^x=\frac{-5y+20}{5}$ **OR** $2^x=-y+4$	✓ 2^x common factor ✓ answer (2)
1.2.2	If $y = -4$, $2^x+2^{x+2}=-5y+20$ $2^x+2^{x+2}=40$ $2^x(1+2^2)=40$ $2^x=8$ $2^x=2^3$ $x=3$	✓ substitution ✓ answer (2)
1.2.3	$-y+4>0$ $y<4$ Largest integer value of y is 3 $2^x=-3+4$ $2^x=1$ $x=0$	✓ $-y+4>0$ ✓ $y=3$ ✓ $x=0$ (3) **[21]**

QUESTION 2

2.1.1	$r=-\frac{32}{64}=-\frac{1}{2}$ $p=256\left(-\frac{1}{2}\right)$ $p=-128$	✓ $-\frac{1}{2}$ ✓ substitution ✓ answer (3)
	OR	
	$\frac{p}{256}=\frac{64}{p}$ $p^2=16384$ $p=\pm128$ $p=-128$	✓ $\frac{p}{256}=\frac{64}{p}$ ✓ $p=\pm128$ ✓ answer (3)
	OR	
	$\frac{p}{256}=\frac{-32}{64}$ $64p=8192$ $p=-128$	✓ $\frac{p}{256}=\frac{-32}{64}$ ✓ simplification ✓ answer (3)
	OR	
	$\frac{1}{r}=\frac{64}{-32}=-2$ $p=-2\times64$ $p=-128$	✓ $\frac{1}{r}=\frac{64}{-32}=-2$ ✓ simplification ✓ answer (3)
2.1.2	$S_n=\frac{a[1-r^n]}{1-r}$ $S_8=\frac{256\left[1-\left(-\frac{1}{2}\right)^8\right]}{1+\frac{1}{2}}$ $=\frac{512}{3}\left(\frac{255}{256}\right)$ $=170$ **OR**	✓ formula ✓ substitution ✓ answer (3)

	$S_n = \dfrac{a\left[1-r^n\right]}{1-r}$ $S_8 = \dfrac{2^8\left[1-\left(-\frac{1}{2}\right)^8\right]}{1+\frac{1}{2}}$ $= \dfrac{2^9}{3}\left(\dfrac{255}{2^8}\right)$ $= 170$	✓ formula ✓ substitution ✓ answer (3)
2.1.3	$-1 < r < 1$ **OR** The common ratio is $-\dfrac{1}{2}$ which is between – 1 and 1. **OR** $-1 < -\dfrac{1}{2} < 1$	✓ answer (1) ✓ answer (1) ✓ answer (1)
2.1.4	$S_\infty = \dfrac{a}{1-r}$ $= \dfrac{256}{1-\left(-\frac{1}{2}\right)}$ $= \dfrac{512}{3}$ $= 170{,}67$	✓ formula ✓ substitution ✓ answer (3)

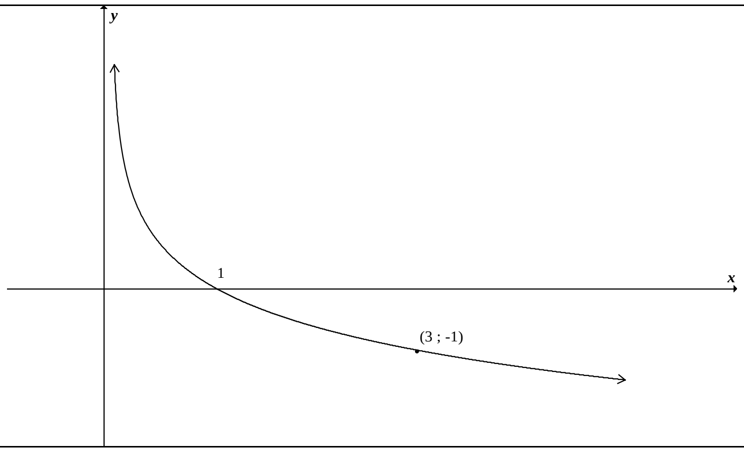
2.2.1	16	✓ answer (1)
2.2.2	$T_n = -8 + 6(n-1)$ $148 = 6n - 14$ $6n = 162$ $n = 27$	✓ substitution into equation ✓T_n = 148 ✓answer (3)
2.2.3	$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$ $\frac{n}{2}\left[2(-8) + (n-1)(6)\right] > 10\,140$ $3n^2 - 11n > 10\,140$ $3n^2 - 11n - 10\,140 > 0$ $(3n + 169)(n - 60) > 0$ When n = 60, S_n = 10 140 Smallest n = 61	✓ $\frac{n}{2}\left[2(-8) + (n-1)(6)\right]$ ✓ $3n^2 - 11n > 10\,140$ ✓ factors ✓ n = 60 ✓ answer (5)
2.3	$\sum_{k=1}^{30}(3k+5)$ $a = 8 \quad n = 30 \quad d = 3$ $\sum_{k=1}^{30}(3k+5) = \frac{30}{2}[2(8) + 29(3)]$ $= 15(103)$ $= 1545$	✓ n = 30 ✓ substitution into correct formula ✓ answer (3) **[22]**

QUESTION 3

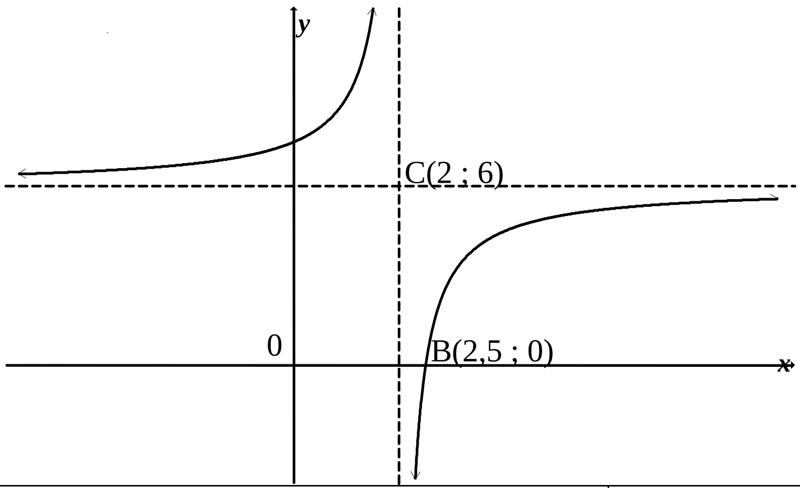
3.1	Jacob calculated that the sequence is geometric or exponential. Vusi calculated that the sequence is quadratic. **OR** Jacob has multiplied each term by 3 to get the next term. Vusi sees it as a sequence with a constant second difference. **OR** Jacob calculated that the sequence is geometric or exponential. Vusi calculated that the sequence can be seen as a combination of exponential and cubic sequences.	✓Jacob (geometric/exponential) ✓ Vusi (quadratic) (2) ✓ Jacob (multiplied each term by 3) ✓ Vusi (constant second difference) (2) ✓Jacob (geometric/exponential) ✓ Vusi (exponential and cubic combined) (2)
3.2.1	$T_n = 3^n$ **OR** $T_n = 3.3^{n-1}$	✓answer (1) ✓answer (1)
3.2.2	3 9 27 57 6 18 30 12 12 $2a = 12$ $3a + b = 6$ $a + b + c = 3$ $a = 6$ $18 + b = 6$ $6 - 12 + c = 3$ $b = -12$ $c = 9$ $T_n = 6n^2 - 12n + 9$ **OR** $2a = 12$ $a = 6$ $T_0 = c = 9$ $T_n = an^2 + bn + 9$ $3 = 6(1)^2 + b(1) + 9$ $b = -12$ $T_n = 6n^2 - 12n + 9$ **OR**	✓ $a = 6$ ✓ method ✓ $b = -12$ ✓ $c = 9$ (4) ✓ $a = 6$ ✓ $c = 9$ ✓ method ✓ $b = -12$ (4)

	$2a = 12$ $a = 6$ $T_n = 6n^2 + bn + c$ $T_1 = 3 = 6(1)^2 + b(1) + c$ i.e. $3 = 6 + b + c$ $T_2 = 9 = 6(2)^2 + b(2) + c$ i.e. $9 = 24 + 2b + c$ $6 = 18 + b$ $b = -12$ $c = 9$ $T_n = 6n^2 - 12n + 9$	✓ $a = 6$ ✓ method ✓ $b = -12$ ✓ $c = 9$ (4)
	OR $T_n = 3^n + k(n-1)(n-2)(n-3)$ $57 = 3^4 + k(3)(2)(1)$ $6k = -24$ $k = -4$ $T_n = 3^n - 4(n-1)(n-2)(n-3)$	✓✓ $T_n = 3^n + k(n-1)(n-2)(n-3)$ ✓ substitution ✓ answer (4) **[7]**

QUESTION 4

4.1	**R** **OR** $(-\infty;\infty)$	✓answer (1)
4.2	$y=0$	✓ $y=0$ (1)
4.3	$x=\left(\frac{1}{3}\right)^{y}$ $y=\log_{\frac{1}{3}} x$ **OR** $x=\left(\frac{1}{3}\right)^{y}$ $x=3^{-y}$ $-y=\log_{3} x$ $y=-\log_{3} x$	✓ $x=\left(\frac{1}{3}\right)^{y}$ ✓ $y=\log_{\frac{1}{3}} x$ (2) ✓ $x=\left(\frac{1}{3}\right)^{y}$ ✓ $y=-\log_{3} x$ (2)
4.4	 	✓ shape ✓intercept at (1 ; 0) ✓any other correct point (3)
4.5	$x=-2$	✓✓ $x=-2$ (2)
4.6	$\text{LHS}=[f(x)]^2-[f(-x)]^2$ $=\left[\left(\frac{1}{3}\right)^{x}\right]^2-\left[\left(\frac{1}{3}\right)^{-x}\right]^2$ $=3^{-2x}-3^{2x}$ $\text{RHS}=f(2x)-f(-2x)$ $=\left(\frac{1}{3}\right)^{2x}-\left(\frac{1}{3}\right)^{-2x}$ $=3^{-2x}-3^{2x}$ $\therefore$ LHS = RHS $[f(x)]^2-[f(-x)]^2=f(2x)-f(-2x)$	✓ $\left[\left(\frac{1}{3}\right)^{x}\right]^2-\left[\left(\frac{1}{3}\right)^{-x}\right]$ ✓ $3^{-2x}-3^{2x}$ ✓ $\left(\frac{1}{3}\right)^{2x}-\left(\frac{1}{3}\right)^{-2x}$ (3) **[12]**

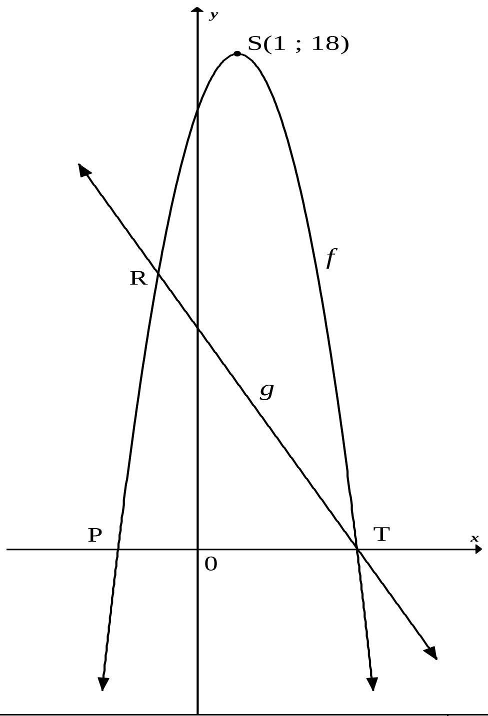
QUESTION 5



5.1	$g(x)=\frac{a}{x-2}+6$ $0=\frac{a}{2{,}5-2}+6$ $0=2a+6$ $a=-3$ $g(x)=\frac{-3}{x-2}+6$	✓ $p=2$ ✓ $q=6$ ✓ substitute B(2,5 ; 0) ✓ $a=-3$ (4)
5.2	$x_f=2-\frac{1}{2}$ $x_f=\frac{3}{2}$ $y_f=6+6$ $y_f=12$ $\text{F}\left(\frac{3}{2};12\right)$	✓ x-coordinate ✓ y-coordinate (2) **[6]**

QUESTION 6

$f(x) = ax^2 + bx + c$
$g(x) = -2x + 8$



6.1	$0 = -2x + 8$ $2x = 8$ $x = 4$ T (4 ; 0)	✓ $y = 0$ ✓ $x = 4$ (2)
6.2	By symmetry, P(– 2 ; 0) $f(x) = a(x+2)(x-4)$ $18 = a(1+2)(1-4)$ $a = -2$ $f(x) = -2(x+2)(x-4)$ $= -2(x^2 - 2x - 8)$ $= -2x^2 + 4x + 16$ **OR** $f(x) = a(x-1)^2 + 18$ $0 = a(4-1)^2 + 18$ $a = -2$ $f(x) = -2(x-1)^2 + 18$ $= -2(x^2 - 2x + 1) + 18$ $= -2x^2 + 4x + 16$	✓ $f(x) = a(x+2)(x-4)$ ✓ substitutes S(1 ; 18) ✓ $a = -2$ ✓ multiplies out correctly to get $-2x^2 + 4x + 16$ (4) ✓ $f(x) = a(x-1)^2 + 18$ ✓ substitutes T(4 ; 0) ✓ $a = -2$ ✓ multiplies out correctly to get $-2x^2 + 4x + 16$ (4)

6.3	$-2x+8=-2x^2+4x+16$ $2x^2-6x-8=0$ $x^2-3x-4=0$ $(x-4)(x+1)=0$ $x=4$ or $x=-1$ at R, $y=-2(-1)+8=10$ i.e. R(– 1; 10)	✓ $-2x+8=-2x^2+4x+16$ ✓ $2x^2-6x-8=0$ ✓ $x=-1$ ✓ $y=10$ (4)
6.4.1	$-1\le x\le 4$	✓ $-1\le x$ ✓ $x\le 4$ (2)
6.4.2	$-2x^2+4x-2<0$ $-2x^2+4x-2+18<18$ $-2x^2+4x+16<18$ $f(x)<18$ $(-\infty\,;1)\cup(1\,;\infty)$ **OR** $-2x^2+4x-2<0$ $-2x^2+4x-2+18<18$ $-2x^2+4x+16<18$ $f(x)<18$ $x\in\mathbf{R}\,;\ x\ne 1$	✓ $-2x^2+4x-2+18<18$ ✓ $-2x^2+4x+16<18$ ✓ $f(x)<18$ ✓ $(-\infty\,;1)\cup(1\,;\infty)$ (4) ✓ $-2x^2+4x-2+18<18$ ✓ $-2x^2+4x+16<18$ ✓ $f(x)<18$ ✓ $x\in\mathbf{R}\,;\ x\ne 1$ (4) **[16]**

QUESTION 7

7.1	$F = P(1+i)^n$ $= 4\,000\,000(1+0{,}06)^3$ $= R4\,764\,064$	✓ formula ✓ substitution ✓ answer (3)
7.2.1	$4\,000\,000 = \dfrac{30\,000\left[1-\left(1+\frac{0{,}06}{12}\right)^{-n}\right]}{\frac{0{,}06}{12}}$ $\dfrac{4000000\times\left(\frac{0{,}06}{12}\right)}{30000} = 1-\left(1+\frac{0{,}06}{12}\right)^{-n}$ $\frac{1}{3} = \left(1+\frac{0{,}06}{12}\right)^{-n}$ $\log_{\left(1+\frac{0{,}06}{12}\right)}\frac{1}{3} = -n$ $n = 220{,}27$ Therefore she will make 220 withdrawals of R30 000. **OR** $4\,000\,000 = \dfrac{30\,000\left[1-\left(1+\frac{0{,}06}{12}\right)^{-n}\right]}{\frac{0{,}06}{12}}$ $\dfrac{4000000\times\left(\frac{0{,}06}{12}\right)}{30000} = 1-\left(1+\frac{0{,}06}{12}\right)^{-n}$ $\frac{1}{3} = \left(1+\frac{0{,}06}{12}\right)^{-n}$ $\log\frac{1}{3} = -n\log\left(1+\frac{0{,}06}{12}\right)$ $n = 220{,}27$ Therefore she will make 220 withdrawals of R30 000.	✓ formula ✓ $i = \frac{0{,}06}{12}$ ✓substitution into correct formula ✓ $\frac{1}{3} = \left(1+\frac{0{,}06}{12}\right)^{-n}$ ✓correct use of logs ✓answer of 220 withdrawals (6) ✓ formula ✓ $i = \frac{0{,}06}{12}$ ✓substitution into correct formula ✓ $\frac{1}{3} = \left(1+\frac{0{,}06}{12}\right)^{-n}$ ✓correct use of logs ✓answer of 220 withdrawals (6)

7.2.2	$4\,000\,000 = \dfrac{20\,000\left[1-\left(1+\frac{0,06}{12}\right)^{-n}\right]}{\frac{0,06}{12}}$ $0 = \left(1+\frac{0,06}{12}\right)^{-n}$ She can make as many withdrawals as she pleases.	✓ $4\,000\,000 = \dfrac{20\,000\left[1-\left(1+\frac{0,06}{12}\right)^{-n}\right]}{\frac{0,06}{12}}$ ✓ $0 = \left(1+\frac{0,06}{12}\right)^{-n}$ ✓ conclusion (3) **[12]**

QUESTION 8

	$\left(1+\frac{0,08}{12}\right)^{12} = \left(1+\frac{r}{2}\right)^{2}$ $\frac{r}{2} = 0,040672622$ $r = 8,13452446\%$ $r = 8,13\%$	✓ $\left(1+\frac{0,08}{12}\right)^{12}$ ✓ $\left(1+\frac{i}{2}\right)^{2}$ ✓ answer **[3]**

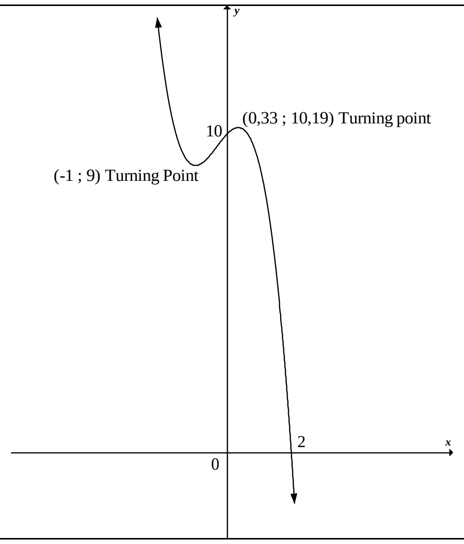
QUESTION 9

9.1	$f(x)=2x^3$ $f(x+h)=2(x+h)^3$ $=2(x^3+3x^2h+3xh^2+h^3)$ $=2x^3+6x^2h+6xh^2+2h^3$ $f(x+h)-f(x)=2x^3+6x^2h+6xh^2+2h^3-2x^3$ $=6x^2h+6xh^2+2h^3$ $f'(x)=\lim_{h\to 0}\frac{6x^2h+6xh^2+2h^3}{h}$ $=\lim_{h\to 0}\frac{h(6x^2+6xh+2h^2)}{h}$ $=\lim_{h\to 0}(6x^2+6xh+2h^2)$ $f'(x)=6x^2$ **OR** $f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ $=\lim_{h\to 0}\frac{2(x+h)^3-2x^3}{h}$ $=\lim_{h\to 0}\frac{2(x^3+3x^2h+3xh^2+h^3)-2x^3}{h}$ $=\lim_{h\to 0}\frac{6x^2h+6xh^2+2h^3}{h}$ $=\lim_{h\to 0}\frac{h(6x^2+6xh+2h^2)}{h}$ $=\lim_{h\to 0}(6x^2+6xh+2h^2)$ $f'(x)=6x^2$	✓ substitution ✓ expansion ✓ formula ✓ $6x^2+6xh+2h^2$ ✓ answer (5) ✓ formula ✓ substitution ✓ expansion ✓ $6x^2+6xh+2h^2$ ✓ answer (5)
9.2	$y=\frac{2\sqrt{x}+1}{x^2}$ $=2x^{-\frac{3}{2}}+x^{-2}$ $\frac{dy}{dx}=-3x^{-\frac{5}{2}}-2x^{-3}$	✓ $2x^{-\frac{3}{2}}$ ✓ x^{-2} ✓ $-3x^{-\frac{5}{2}}$ ✓ $-2x^{-3}$ (4)

9.3	$f'(-1) = -7$ $f'(x) = 2ax + b$ $-7 = -2a + b$ $f(-1) = -7(-1) + 3$ $= 10$ $\therefore\ a - b + 5 = 10$ $a - b = 5............[1]$ $-2a + b = -7.........[2]$ $-a = -2.........[1] + [2]$ $a = 2$ $b = -3$	✓ $f'(x) = 2ax + b$ ✓ substitution of $x = -1$ ✓ $-7 = -2a + b$ ✓ $f(-1) = 10$ ✓ $a = 2$ ✓ $b = -3$ (6) **[15]**

QUESTION 10

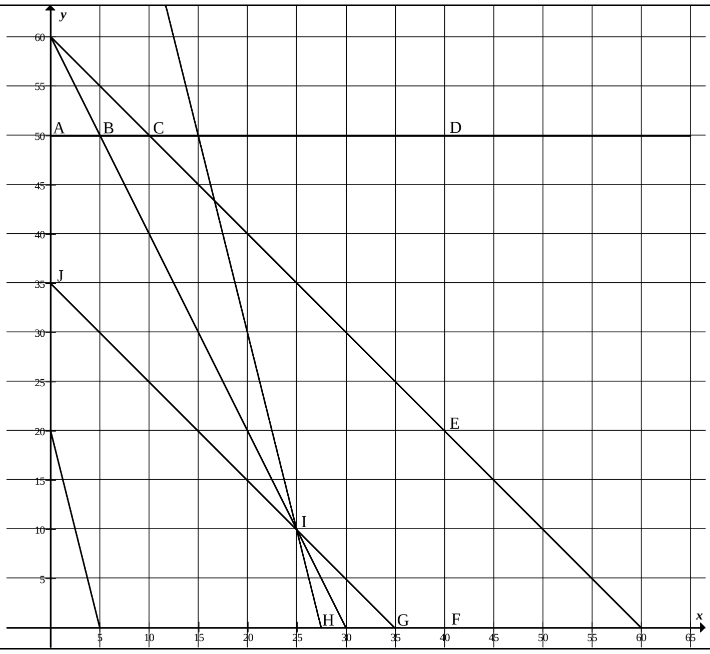
$$f(x) = -x^3 - x^2 + x + 10$$

10.1	$(0;10)$	✓ $(0;10)$ (1)
10.2	$0 = -x^3 - x^2 + x + 10$ $0 = -(x-2)(x^2+3x+5)$ $x - 2 = 0$ or $x^2 + 3x + 5 = 0$ $x = 2$ $x = \frac{-3 \pm \sqrt{3^2 - 4(1)(5)}}{2(1)}$ $= \frac{-3 \pm \sqrt{-11}}{2}$ which has no solution Therefore the only x-intercept of f is $(2;0)$	✓ $(x-2)$ ✓ (x^2+3x+5) ✓ $x = \frac{-3 \pm \sqrt{-11}}{2}$ ✓ no solution (4)
10.3	$f'(x) = -3x^2 - 2x + 1$ $0 = -3x^2 - 2x + 1$ $0 = (3x-1)(x+1)$ $x = \frac{1}{3}$ or $x = -1$ $y = -\left(\frac{1}{3}\right)^3 - \left(\frac{1}{3}\right)^2 + \left(\frac{1}{3}\right) + 10$ or $y = -(-1)^3 - (-1)^2 + (-1) + 10$ $= \frac{275}{27}$ $= 9$ $\left(\frac{1}{3}; 10\frac{5}{27}\right)$ $(-1; 9)$	✓ $f'(x) = -3x^2 - 2x + 1$ ✓ $f'(x) = 0$ ✓ factors ✓ x-values ✓ $\left(\frac{1}{3}; 10\frac{5}{27}\right)$ ✓ (– 1 ; 9) (6)
10.4		✓ shape ✓ intercepts ✓ turning points (3) **[14]**

QUESTION 11

11.1	Length of box = $3x$ Volume $= l \times b \times h$ $9 = 3x \cdot x \cdot h$ $9 = 3x^2 h$ $h = \frac{3}{x^2}$	✓ length of box = $3x$ ✓ $9 = 3x \cdot x \cdot h$ ✓ $h = \frac{3}{x^2}$ (3)
11.2	$C = (2(3xh) + 2xh) \times 50 + (2 \times 3x^2) \times 100$ $= 8x\left(\frac{3}{x^2}\right) \times 50 + 600x^2$ $= \frac{1200}{x} + 600x^2$ **OR** $C = (h \times 8x) \times 50 + (2 \times 3x^2) \times 100$ $= 8x\left(\frac{3}{x^2}\right) \times 50 + 600x^2$ $= \frac{1200}{x} + 600x^2$	✓ $(2(3xh) + 2xh) \times 50$ ✓ $(2 \times 3x^2) \times 100$ ✓ substitution of $h = \frac{3}{x^2}$ (3) ✓ $(h \times 8x) \times 50$ ✓ $(2 \times 3x^2) \times 100$ ✓ substitution of $h = \frac{3}{x^2}$ (3)
11.3	$C = 1200x^{-1} + 600x^2$ $\frac{dC}{dx} = -1200x^{-2} + 1200x$ $0 = -1200x^{-2} + 1200x$ $1200x^3 = 1200$ $x^3 = 1$ $x = 1$ Therefore the width of the box is 1 metre.	✓ $\frac{dC}{dx} = -1200x^{-2} + 1200x$ ✓ $\frac{dC}{dx} = 0$ ✓ $x^3 = 1$ ✓ $x = 1$ (4) **[10]**

QUESTION 12

12.1		✓✓ region ABIJ shaded (2) **NOTE:** If region BCEFGI is shaded: award ONE mark If any other region is shaded: award 0 marks
12.2	$x \leq 40$ $x + y \leq 60$ $y \geq 0$	✓ $x \leq 40$ ✓ $x + y \leq 60$ ✓ $y \geq 0$ (3)
12.3	$x = 25$	✓answer (1)
12.4	At I(25 ; 10), $P = 4(25) + 10 = 110$ Maximum value of P is 110 when $x = 25$ and $y = 10$	✓ $x = 25$ ✓ $y = 10$ ✓substitution ✓ maximum value of P is 110 (4)
12.5	$C = kx + y$ $y = -kx + C$ $-k < -1$ $k > 1$	✓ $y = -kx + C$ ✓ $k > 1$ (2) **NOTE:** Answer only: award TWO marks **[12]**

TOTAL: 150